Grade 12 Math

MEETING THE EXPECTATIONS

BAY STREET DERIVATIVES

ALL COOPED UP

ROLLER COASTER FUNCTIONS

SUMMATIVE ASSESSMENT

ANSWER KEY

Welcome Grade 12 Teachers to
Canada's Wonderland's Math Program!

We have provided you with activities that will take you from your classroom to an action filled day at the Park. The **BEFORE THE PARK** activities are set up for your students to practice some new skills and review some old ones before they go to the Park. The **AT THE PARK** activities are a continuation and extension of the classroom activities. The tasks set up for your students at the Park are designed to let them enjoy all that Canada's Wonderland has to offer, while gathering some data to be used back at the school. The students will use this information to complete a **SUMMATIVE ASSESSMENT** that allows them to extend the experiences that they began in the classroom before the trip. Every activity is completely linked to the new revised Mathematics Curriculum.

Every activity is designed as a real-world experience. As in the real world, there are many possible solutions to a variety of questions. We encourage you to challenge your students to think deeply and reflect on the tasks that are set out before them. We hope that this experience will be a celebration and extension of your teaching and learning this year.

Thank you for your on-going support for young people and our programs at Canada's Wonderland.

MEETING THE EXPECTATIONS

A Correlation with the Ontario Mathematics Curriculum 12 (Calculus)

Activity	Expectations
BAY STREET DERIVATIVES	• describe examples of real-world applications of rates of change, represented in a variety of ways • compare the calculation of instantaneous rates of change at a point (a, $f(a)$) for polynomial functions • verify the power rule for functions of the form $f(x) = x$, where n is a natural number • verify the constant, constant multiple, sum, and difference rules numerically • determine algebraically the derivatives of polynomial functions, and use these derivatives to determine the instantaneous rate of change at a point • recognize the second derivative as the rate of change of the rate of change • determine algebraically the equation of the second derivative $f''(x)$ of a polynomial function $f(x)$ • make connections between the concept of motion (i.e., displacement, velocity, acceleration) and the concept of the derivative • solve problems, using the derivative that involve instantaneous rates of change
ALL COOPED UP	• describe examples of real-world applications of rates of change, represented in a variety of ways • compare the calculation of instantaneous rates of change at a point (a, $f(a)$) for polynomial functions • verify the power rule for functions of the form $f(x) = x$, where n is a natural number • verify the constant, constant multiple, sum, and difference rules numerically • determine algebraically the derivatives of polynomial functions, and use these derivatives to determine the instantaneous rate of change at a point • recognize the second derivative as the rate of change of the rate of change • determine algebraically the equation of the second derivative $f''(x)$ of a polynomial function $f(x)$ • make connections between the concept of motion (i.e., displacement, velocity, acceleration) and the concept of the derivative • solve problems, using the derivative that involve instantaneous rates of change

ROLLER COASTER FUNCTIONS	• compare, through investigation, the calculation of instantaneous rates of change at a point (*a*, *f*(*a*)) for polynomial functions • determine numerically and graphically the intervals over which the instantaneous rate of change is positive, negative, or zero for a function that is smooth over these intervals • verify the power rule for functions of the form $f(x) = x$, where n is a natural number • verify the constant, constant multiple, sum, and difference rules graphically and numerically • determine algebraically the derivatives of polynomial functions, and use these derivatives to determine the instantaneous rate of change at a point • make connections, through investigation using technology, between the key features of the graph of the function (e.g., increasing/ decreasing intervals, local maxima and minima, points of inflection, intervals of concavity) • describe key features of a polynomial function, given information about its first and/or second derivatives • sketch the graph of a polynomial function, given its equation, by using a variety of strategies • solve problems, using the derivative, that involve instantaneous rates of change • solve optimization problems involving polynomial functions
SUMMATIVE ASSESSMENT Authentic Construction and Design Inquiry	• describe connections between the average rate of change of a function that is smooth • compare, through investigation, the calculation of instantaneous rates of change at a point (*a*, *f*(*a*)) for polynomial functions • verify the power rule for functions of the form $f(x) = x$, where n is a natural number • verify the constant, constant multiple, sum, and difference rules graphically and numerically determine algebraically the derivatives of polynomial functions, and use these derivatives to determine the instantaneous rate of change at a point • sketch the graph of a derivative function, given the graph of a function that is continuous over an interval, and recognize points of inflection of the given function • sketch the graph of a polynomial function, given its equation, by using a variety of strategies • solve problems, using the derivative, that involve instantaneous rates of change

Before the Park

This activity will engage you in activities designed to prepare you to perform similar calculations on Drop Tower during your trip to Canada's Wonderland.

1. If a ball is dropped from the top of TD tower, 220 meters, then its height in meters after *t* seconds is given by

$$h = 220 - 4.9t^2$$

a.) The velocity of an object is defined as the rate of change of its position. That is the derivative of the height function gives the velocity. Find the velocity of the ball after:

i) 1 second

ii) 2 seconds

iii) 3 seconds

b) The acceleration of a falling object is defined as the rate of change of its velocity. What is the acceleration of the falling ball?

c) What do you know about your answer in part b? If its derivative were taken what would be your result?

At the Park

Canada's Wonderland contains many rides that have been designed to work within pre-determined parameters that are defined by math and physics. In this activity you will find the maximum speed of passengers riding Drop Tower.

1. Find the sign indicating the base distance to Drop Tower.

   distance =_________m

2. Using your horizontal accelerometer calculate the height of Drop Tower

$\tan\theta = h_1 / L$, $h_1 = L\tan\theta$; h_2 = height of eye from ground; h = total height of ride = $h_1 + h_2$

3. The height of the passenger compartment, (in meters), as it falls, is given by $h=x-4.9t^2$ where x is the height of the tower you calculated in Question 2 and t is the time in seconds.
   Ride Drop Tower and measure the time that it takes from initial release at the top of the ride until you feel the ride slowing down. $t=$ _____ *s*

4. Using the height of Drop Tower that you calculated in question 2 calculate the velocity of the passengers on Drop Tower at the time you measured in question 3.

5. Calculate the acceleration of Drop Tower. What do you notice about its value? Compare this to the ball falling in the Before the Park activity.

Before the Park

The distance travelled by a car is given by $s(t) = 40t^2 + 20t$ where t is measured in hours and s is measured in kilometres.

1. The velocity of an object is defined as the rate of change of its position. Find the velocity of the car after:

i) half an hour

ii) one hour

2. After what time did the velocity reach 120 km/hr

3. Acceleration is defined as the rate of change of velocity. What is the acceleration of the car after half an hour?

At the Park

Back Lot Stunt Coaster was opened at Canada's Wonderland on May 1, 2005. The ride features unique cars which resemble ¾ scale MINI Cooper vehicles. Unlike other coasters no hill is needed as the cars gain speed twice during the ride through magnetic linear induction motors. Special effects are incorporated throughout the ride, such as sound effects built into the cars, a helecopter that attacks riders with simulated machine gun sound, a fire, and pyrotechnic and water effects.

1. Ride Back Lot Stunt Coaster and measure the time for the launch before the first turn and again during the second launch mid way through the ride.

   (First launch) t_1=_____*seconds* (Second launch) t_2=_____*seconds*

The distance travelled by the cars during the launches is given by $s(t)=3t^2 + t$

2. What is the maximum velocity of the cars after they reach the end of launch 1 and launch 2?

3. What is the acceleration of the cars during each of the launches?

Before the Park

The position of a roller coaster, at time t, in seconds, moving along a line is given by

$$s(t) = 2t^3 - 30t^2 + 120t \quad \text{for } 0 \le t \le 10$$

1. Determine the velocity and acceleration of the roller coaster at any time t

2. At what time(s) is the roller coaster stopped?

3. When is the roller coaster moving at a steady speed (when is its velocity not changing)?

4. At what time(s) is the roller coaster speeding up (when is its velocity increasing)?

5. At what time(s) is it slowing down (when is its velocity decreasing)?

At the Park

Travel through the park and find different rides at Canada's Wonderland that mirror the following parts of a graph; concave up, concave down, local minimum, local maximum, absolute minimum, absolute maximum, vertical asymptotes, and inflection point:

In the boxes below:

i) state which ride you chose and *where* on the ride the description occurs
ii) make a sketch to clearly show the description
iii) give an explanation as to why this description occurs here

Name Ride 1:___________________ **Roller Coaster Track is *Concave Up***	**Name Ride 2:____________________** **Roller Coaster Track is *Concave Down***
Name Ride 3:___________________ **Roller Coaster Track has a *Local Minimum***	**Name Ride 4:____________________** **Roller Coaster Track has a *Local Maximum***

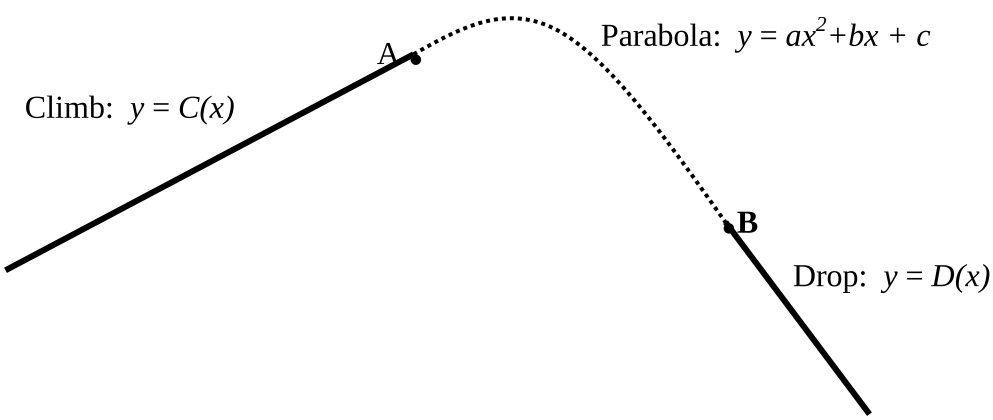
Name Ride 5:__________________ **Roller Coaster Track has an *Absolute Minimum***	**Name Ride 6:____________________** **Roller Coaster Track has an *Absolute Maximum***
Name Ride 7:___________________ **Vertical Asymptote**	**Name Ride 8:___________________** **Point of Inflection**

SUMMATIVE ASSESSMENT

Authentic Construction and Design Inquiry

A new roller coaster arrives at Canada's Wonderland and is missing the parabolic section joining the first climb to the first drop. The production deadline is looming and engineers at the fabrication plant need you to send specifications for this new part as soon as possible.

Given that the first climb and drop sections are straight; the slope of the first climb is 0.6, and the slope of the first drop is -1.4. In between these sections, a parabolic portion of track will be inserted; the transition points A and B must be smooth (ie.,the straight sections are tangent to the ends of the parabola). Determine the function that describes the parabola needed to rectify this problem by following the steps below.



1. If $y = f(x) = ax^2+bx + c$ represents the parabola portion of the track, write out the derivative of $f(x)$ with respect to x. This derivative represents the rate of change of the parabola at any point x.

2. The straight sections of track must be smooth at the transition points A and B. If the horizontal distance between A and B is 30 meters, find the coefficients *a*, *b* and *c* and hence the equation of the parabola. Choose the origin (0, 0) to be at point **A** for simplicity.
   Hint: The slope of the climb is $f'(0)$ and the slope of the drop is $f'(30)$.

3. What is the equation of the line that describes the first Climb?

4. a) What is the *y – coordinate* at point B?

b) With this value of the *y – coordinate*, find the equation of the line that describes the first Drop.

5. What is the difference in elevation between points A and B?